Assignment 1

Factorial (Two-Way) ANOVA

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You will be able to perform a factorial analysis of variance (ANOVA) and interpret the meaning of the results (both main effects and interactions).

Variables (Revisited)

- Independent Variables
  - a.k.a. Factors

- Dependent Variables
  - a.k.a. Measures

Factor

- e.g., technique:
  - TreeMap vs. Phylotrees vs. ArcTrees

- How many *levels* does the "technique" factor have?

Factorial Design

- Remember the assignment
- Two factors:
  - gender (male, female)
  - technique (TreeMap, Phylotrees, ArcTrees)

Factorial Design

Factors

Levels

		Gender	
		Male	Female
Technique	TreeMap	Group 1	Group 2
	Phylotrees	Group 3	Group 4
	ArcTrees	Group 5	Group 6

Cells

How did we test these six groups?

- What did we find out?
- What could we have found out?
- Was what we did valid?

Factorial ANOVA

- What is the null hypothesis?

Factorial ANOVA

- What are the null hypotheses?

Main Effects

Main Effects

		Gender		
		Male	Female	
Technique	TreeMap	μ_1	μ_2	$\mu_{TreeMap}$
	Phylotrees	μ_3	μ_4	$\mu_{Phylotrees}$
	ArcTrees	μ_5	μ_6	$\mu_{ArcTrees}$
		μ_{Male}	μ_{Female}	

Null Hypotheses

- Main effect of *gender*:

$$\mu_{\text{Male}} = \mu_{\text{Female}}$$

- Main effect of *technique*:

$$\mu_{\text{TreeMap}} = \mu_{\text{Phylotrees}} = \mu_{\text{ArcTrees}}$$

Example

- Do headlights help see pedestrians?

		Time of Day	
		Night	Day
Lights	Headlights	μ_1	μ_2
	No Headlights	μ_3	μ_4

Example

- Do headlights help see pedestrians?

Lights	Time of Day: Night	Time of Day: Day
Headlights	*good*	*good*
No Headlights	*bad*	*good*

Results?

- Main effects
  - Day is “better” than night
  - Headlights are “better” than no headlights
- Is that the real story?

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What about the cell means?

- What other null hypothesis could we test?

$$\mu_1 = \mu_2 = \ldots = \mu_4$$

- Why not?

Interactions

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What is our third null hypothesis?

- e.g. conclusion:
  - The effect of headlights depends on whether it is day or night.

- Null hypothesis in words:
  - The main effect of one factor does *not* depend on the levels of another factor.

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Alternative Hypothesis

- The main effect of factor *X* depends on the levels of factor *Y*.

Null Hypothesis

		Factor 1	
		Level 1	**Level 2**
Factor 2	**Level 1**	μ_{11}	μ_{12}
	Level 2	μ_{21}	μ_{22}
	Level 3	μ_{31}	μ_{32}

Null Hypothesis

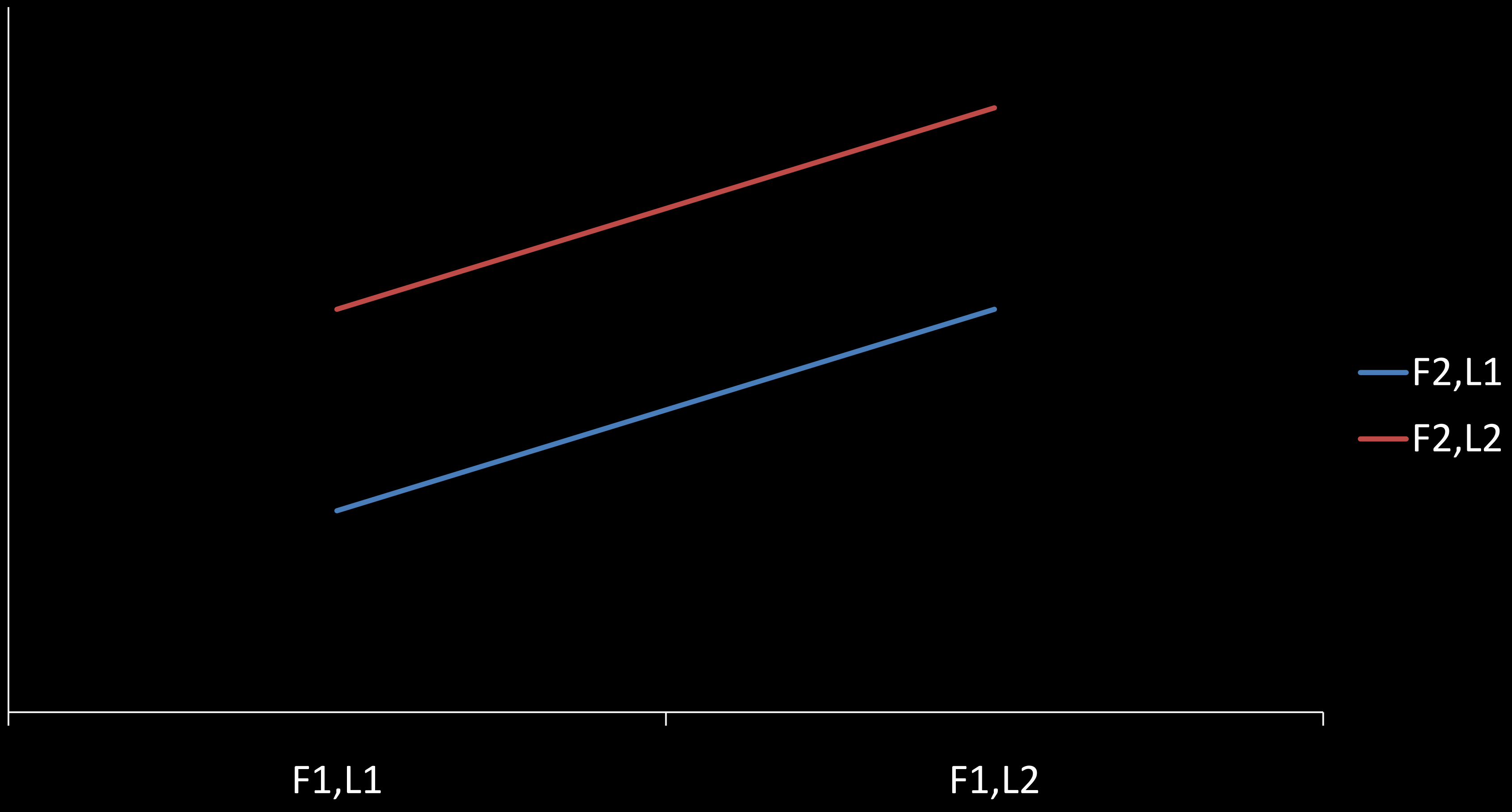
$$\mu_{11} - \mu_{12} = \mu_{21} - \mu_{22} = \mu_{31} - \mu_{32}$$

- In general:

$$\mu_{ij} - \mu_{i'j} = \mu_{ij'} - \mu_{i'j'}$$

for all combinations of i, i’, j, j’

Null Hypothesis



Null Hypotheses

- Main effects:
  - row means are equal
  - column means are equal

- Interaction:
  - the pattern of differences in one row/column do not account for the pattern of differences in another row/column

Factorial ANOVA Math

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F-scores

- Calculate F for each null hypothesis

$$F = \frac{MS_{BG}}{MS_{WG}}$$

Sum of Squares (revisited)

$$SS = \sum_{i=1}^{n} (x_i - \mu)^2$$

$$= \sum_{i=1}^{n} (x_i)^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2$$

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Sum of Squares (revisited)

$$SS_{Total} = SS_{BG} + SS_{WG}$$

$$SS_{BG} = SS_{rows} + SS_{cols} + SS_{rows \times cols}$$

Factorial ANOVA Table

	Degrees of Freedom	Sum of Squares	Mean Square	F
Factor 1				
Factor 2				
Interaction				
Within Groups				
Total				

Example

	Degrees of Freedom	Sum of Squares	Mean Square	F
Gender	1	712.89	712.89	6.38
Condition	1	462.25	462.25	4.14
Gender × Condition	1	1.21	1.21	0.01
Error (Within Groups)	96	10,720.82	111.68	
Total	99	11,897.17		

Break: 15 Minutes

Example

raw data		B			
		level 1		level 2	
A	level 1	20.4	17.4	20.5	26.3
		20.0	18.4	26.6	19.8
		24.5	21.0	25.4	28.2
		19.7	22.3	22.6	23.7
		17.3	23.3	22.5	22.6
	level 2	22.4	19.1	34.1	21.9
		22.4	25.4	32.6	28.5
		26.2	25.1	29.0	25.8
		28.8	21.8	29.0	27.1
		26.3	25.2	25.7	24.4

Example

raw data		B: level 1	B: level 2	rows
A	level 1	$N_{g1}=10$ $X_{g1}=204.3$ $X^2_{g1}=4226.3$	$N_{g2}=10$ $X_{g2}=238.2$ $X^2_{g2}=5741.4$	$N_{r1}=20$ $X_{r1}=442.5$
	level 2	$N_{g3}=10$ $X_{g3}=242.7$ $X^2_{g3}=5961.34$	$N_{g4}=10$ $X_{g4}=278.1$ $X^2_{g4}=7855.3$	$N_{r2}=20$ $X_{r2}=520.8$
	columns	$N_{c1}=20$ $X_{c1}=447.0$	$N_{c2}=20$ $X_{c2}=516.3$	$N_T=40$ $X_T=963.3$ $X^2_T=23784.4$

Example

Source	SS	df	MS	F	P
between groups	273.39	1			
rows	153.27	1	153.27	17.67	<.01
columns	120.06	1	120.06	13.84	<.01
interaction	0.06	1	0.06	0.01	ns
within groups (error)	312.31	36	8.68		
TOTAL	585.70	39			

Three-Way ANOVA

What’s different about a three-way ANOVA?

Example

- Factors:
  – Gender (Male, Female)
  – Screen Orientation (Horizontal, Vertical)
  – Screen Size (Small, Medium, Large)

Null Hypotheses

- Main effects
  - gender, screen orientation, screen size (no diff)
- Interactions (2-way)
  - gender × screen orientation
  - gender × screen size
  - screen orientation × screen size (no pattern)
- Interactions (3-way)
  - gender × screen orientation × screen size (?)

Three-way Alternate Hypothesis

- Interpretation #1:
  - The main effect of a factor depends on the levels of both of the other two factors
- Interpretation #2:
  - The interaction effect between two factors depends on the level of another

Factor 1

Factor 2	Level 1	Level 2
Level 1	μ_{111}	μ_{121}
Level 2	μ_{211}	μ_{221}
Level 3	μ_{311}	μ_{321}

Level 1

Factor 1

Factor 2	Level 1	Level 2
Level 1	μ_{112}	μ_{122}
Level 2	μ_{212}	μ_{222}
Level 3	μ_{312}	μ_{322}

Level 2

Factor 3

Factorial ANOVA Table

	Degrees of Freedom	Sum of Squares	Mean Square	F
Factor 1				
Factor 2				
Factor 3				
Factor 1 × Factor 2				
Factor 1 × Factor 3				
Factor 2 × Factor 3				
Factor 1 × Factor 2 × Factor 3				
Within Groups				
Total				

Between-Participants vs. Within-Participants

Participant Assignment

		Factor 1	
		Level 1	Level 2
Factor 2	Level 1	*N* = 10	*N* = 10
	Level 2	*N* = 10	*N* = 10
	Level 3	*N* = 10	*N* = 10

Between Participants

	Factor 1: Level 1	Factor 1: Level 2
Factor 2: Level 1	*Group 1*	*Group 2*
Factor 2: Level 2	*Group 3*	*Group 4*
Factor 2: Level 3	*Group 5*	*Group 6*

Within Participants

		Factor 1	
		Level 1	**Level 2**
Factor 2	**Level 1**	*Group 1*	*Group 1*
	Level 2	*Group 1*	*Group 1*
	Level 3	*Group 1*	*Group 1*

Mixed Design

		Factor 1	
		Level 1	**Level 2**
Factor 2	**Level 1**	*Group 1*	*Group 2*
	Level 2	*Group 1*	*Group 2*
	Level 3	*Group 1*	*Group 2*

How does this change the math?

T-test

- Independent Variance

$$\text{variability} = \sqrt{\frac{\sigma_T^2}{n_T} + \frac{\sigma_P^2}{n_P}}$$

- Paired Variance

One-Way ANOVA

- Independent

	Degrees of Freedom	Sum of Squares	Mean Square	F
Between Groups				
Within Groups				
Total				

- Repeated Measures

	Degrees of Freedom	Sum of Squares	Mean Square	F
Factor				
Subjects				
Error (Factor × Subjects)				
Total				

Two-Way ANOVA

	Degrees of Freedom	Sum of Squares	Mean Square	F
Subjects				
Factor 1				
Factor 1 × Subjects				
Factor 2				
Factor 2 × Subjects				
Factor 1 × Factor 2				
Factor 1 × Factor 2 × Subjects				
Total				

Post-hoc Analysis

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Main Effects

- Main effect for factor with two levels
  - No need to do post-hoc
- Main effect for factor with >2 levels
  - Same as one-way ANOVA
  - Pairwise t-tests

How do we (correctly) interpret the results when there's an interaction effect?

Example

- There is a significant interaction between *gender* and *technique*.

- example answer: men were quicker with TreeMaps than with Phylotrees and ArcTrees, but women were quicker with ArcTrees than with Phylotrees and TreeMaps.

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Example

Post-hoc Tests

- For each level of one factor
  - pairwise comparisons of each level of the other

- Hold *level* of one factor *constant*

Three-Way Interactions

- Much more difficult to interpret
- Same strategy: hold levels of *two* factors constant and perform pairwise comparisons
- Alternate strategy: don't bother (use graphs)